

Goldstein

$$1.3. (1.22): M \frac{d^2 \vec{R}}{dt^2} = \sum_i \vec{F}_i^{(e)} \equiv \vec{F}^{(e)}, \quad (1.26): \frac{d\vec{L}}{dt} = \vec{N}^{(e)}$$

Suppose 2 particles satisfy both, then

$$M \frac{d^2 \vec{R}}{dt^2} = m_1 \frac{d^2 \vec{r}_1}{dt^2} + m_2 \frac{d^2 \vec{r}_2}{dt^2}$$

$$= \vec{F}_1 + \vec{F}_2 = \vec{F}_1^{(e)} + \vec{F}_{21} + \vec{F}_2^{(e)} + \vec{F}_{21}$$

equating this with $\vec{F}^{(e)} = \vec{F}_1^{(e)} + \vec{F}_2^{(e)}$, we have $\vec{F}_{21} + \vec{F}_{12} = 0$.

This is the weak law of action and reaction.

$$\text{Expanding } \frac{d\vec{L}}{dt}, \text{ we have } \frac{d\vec{L}}{dt} = \frac{d}{dt} [\vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2],$$

$$= \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$

$$= \vec{r}_1 \times [\vec{F}_1^{(e)} + \vec{F}_{21}] + \vec{r}_2 \times [\vec{F}_2^{(e)} + \vec{F}_{12}]$$

$$\equiv \vec{r}_1 \times \vec{F}_1^{(e)} + \vec{r}_2 \times \vec{F}_2^{(e)} + \vec{r}_1 \times \vec{F}_{21} + \vec{r}_2 \times \vec{F}_{12}$$

The weak law of action and reaction implies $\vec{F}_{12} = -\vec{F}_{21}$,
so $\vec{r}_1 \times \vec{F}_{21} + \vec{r}_2 \times \vec{F}_{12} = (\vec{r}_1 - \vec{r}_2) \times \vec{F}_{21} = \vec{r}_{12} \times \vec{F}_{21}$.

Equating the above quantity with $\vec{N}^{(e)} = \vec{r}_1 \times \vec{F}_1^{(e)} + \vec{r}_2 \times \vec{F}_2^{(e)}$,

we have the strong law of action and reaction, since $\vec{r}_{12} \times \vec{F}_{21} = 0$ indicates they are aligned.